**Definition:** A type $A$ is contractible, if there is $a : A$, called the center of contraction, such that for all $x : A$, $a = x$.

**Definition:** A map $f : A \to B$ is an equivalence, if for all $y : B$, its fiber, \( \{ x : A \mid f x = y \} \), is contractible. We write $A \simeq B$, if there is an equivalence $A \to B$.

**Lemma:** For each type $A$, the identity map, $1_A := \lambda x : A . x : A \to A$, is an equivalence.

**Proof:** For each $y : A$, let \( \{ y \}_A := \{ x : A \mid x = y \} \) be its fiber with respect to $1_A$ and let $\bar{y} := (y, r_A y) : \{ y \}_A$. As for all $y : A$, $(y, r_A y) = y$, we may apply \( \text{Id-induction} \) on $y, x : A$ and $z : (x = y)$ to get that $(x, z) = y$.

Hence, for $y : A$, we may apply $\Sigma$ -elimination on $u : \{ y \}_A$ to get that $u = y$, so that $\{ y \}_A$ is contractible. Thus, $1_A : A \to A$ is an equivalence. $\Box$

**Corollary:** If $U$ is a type universe, then, for $X, Y : U,$

\[(*) X = Y \to X \simeq Y\]

**Proof:** We may apply the lemma to get that for $X : U$, $X \simeq X$. Hence, we may apply Id-induction on $X, Y : U$ to get that $(*)$. $\Box$

**Definition:** A type universe $U$ is univalent, if for $X, Y : U$, the map $E_{X,Y} : X = Y \to X \simeq Y$ in $(*)$ is an equivalence.